CFD calculations of Küssner-type indicial functions for bluff bodies

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1 Introduction

The indicial response is defined as the transient response of a system to a step input. For aerodynamic applications, such typical responses would be the transient lift response of a body to step change in the characteristics of the incoming flow, such as a step change in the angle of attack or penetration into a sharp edged gust. The aim of this study is to demonstrate that available calculation procedures can be used to obtain aerodynamic indicial functions, commonly used to improve two-dimensional formulations for wind forces on elongated bodies, such as bridge decks. The present research focuses on Küssner-type indicial functions.

2 Buffeting analysis

Buffeting refers to wind induced vibrations due to air-turbulence. Scanlan [2, 3] recalled that "buffeting forces acting on elongated bodies like long-span bridges have been described by theories and analytical formats strongly influenced by analogous expressions found in two-dimenensionnal airfoil theory". Therefore, current theories for prediction of incident turbulent exitations are based upon a quasi-static formulation which relates the buffeting forces to the incoming wind velocity components. In this format the lift is given by

$$L_b(s) = \tfrac{1}{2}\rho U^2 C \quad [2C_l(\alpha_0)\frac{u_x(s)}{U} \quad + \quad C_l'(\alpha_0)\frac{u_z(s)}{U}] \tag{1}$$

Where s is a dimensionless time, $C_l(\alpha_0)$ and $C_l'(\alpha_0)$ the steady lift coefficient and lift slope, C the structure chord length, ρ air density, U the cross flow velocity, u_x and u_z the along-wind and vertical gust velocity components respectively. Analogous formulations can be obtained for drag and moment expressions. They could be improved through the introduction of indicial functions. Following, as a guide method, the thin airfoil theory, the lift may be written in the frequency domain as

$$\hat{L}_b(k) = \tfrac{1}{2}\rho U^2 C \quad [2C_l(\alpha_0)\frac{\hat{u}_x(k)}{U}\hat{\Phi}'_{l_x} \quad + \quad C_l'(\alpha_0)\frac{\hat{u}_z(k)}{U}\hat{\Psi}'_{l_z}] \tag{2}$$

Where $\hat{L}_b(k)$ is the Fourier transform of $L_b(s)$, k is a reduced frequency, $\Phi_{l_x}(k)$ and $\Psi_{l_z}(k)$ are Wagner-type and Küssner-type indicial functions. Wagner (1925) derived the indicial response of a thin airfoil to a step change in angle of attack and Küssner (1936) the loading response to an airfoil penetrating a sharp edged gust. Both Küssner and Wagner used potential flow theory and considered a thin airfoil at low angle of attack motions. The spectral form of (2) is

$$S_{L_b}(k) = \tfrac{1}{4}\rho^2 U^4 C^2 \quad [4C_l^2(\alpha_0)\frac{S_{u_x}}{U^2}\chi^2_{l_x} \quad + \quad C_l'^2(\alpha_0)\frac{S_{u_z}}{U^2}\chi^2_{l_z}] \tag{3}$$

This expression represents a simplification of the complete equation, since the cross power spectral densities of wind turbulence components $S_{u_x u_z}$ and $S_{u_z u_x}$

have been assumed to be negligible (this is a commonly used assumption). In these formulations, $\chi^2_{l_x}(k)$ and $\chi^2_{l_z}(k)$ are so-called "aerodynamic admittance" related to the indicial functions

$$\chi^2_{l_x} = \hat{\Phi}'_{l_x}(k)\hat{\Phi}'^*_{l_x}(k) \qquad \chi^2_{l_z} = \hat{\Psi}'_{l_z}(k)\hat{\Psi}'^*_{l_z}(k)$$

These equations quantitatively define $\chi^2_{l_i}(k) \quad (i = x, z)$ as frequency-based transfer functions, relating the spectrum of incident gusting velocities to that of associated lift. In the past, the Sears function was use as admittance but many authors have shown that this assumption was not appropriate, even for streamlined bridge deck sections. Moreover, in the context of bluff bodies, turbulence is induced by the structure itself (signature turbulence) and aifoil-type theories are unable to depict wind actions. In this case a CFD acquisition of indicial and/or admittance functions could be a step toward predicting the cross-wind excitation induced by the incident wind.

3 Outline of numerical procedure

The present research investigates only indicial lift responses of structures penetrating (or enveloped by) a sharp edged gust. The functional forms of these responses are the so-called Küssner-type indicial functions. To obtain these functions, the wind flows across the respective sections have been computed using a finite-element flow solver, CASTEM 2000.

The wind has been assumed as an incompressible turbulent flow governed, in the atmospheric boundary layer, by the Navier-Stokes equations. CASTEM solved the Reynolds Average Navier-Stokes (RANS) equations. Throughout this study the RNG $k-\varepsilon$ turbulence model has been used with a Reynolds number of 8.10^6 (a previous study [4] has shown that this model was more adapted to unsteady flows, with organised vortices, than the standard $k-\varepsilon$ model).

The variables were rendered nondimensional with regards to the section chord C and the inlet flow velocity U_0. The computational domains used were $4C$ long upstream from the leading edge of the section, $12C$ long dowstream of the section and $4C$ wide on each side of the section. The calculations have been carried out on a a $4 \times 77 \times 46$ grid. The flow with 0^o angle of attack has been first computed with Dirichlet conditions imposed at the inflow boundary (freestream conditions with a turbulence level of 2 %), stress free conditions automatically imposed on the other boundaries (inherently by the finite element formulation) and near wall model on the wall boundary.

Next, the vertical velocity of the flow field upstream of the airfoil was changed to $\frac{u_{z0}}{U_0} = 0,0875$ (5^o angle of attack). The indicial lift response was obtained by integrating the pressure along the boundary of the section as the gust convects with the freestream. The corresponding aerodynamic admittance were obtained by a Fourier transform relationship. This method is based upon the one suggested by Brar [1] to calculate flutter derivatives. It has been tested for a NACA0012 airfoil and carried out for a rectangular bluff section ($\frac{H}{C} = 0.12$) and for a bridge deck (Normandie Bridge).

4 Computed results

For the airfoil, the indicial lift response due to a stationary gust, as a function of time, is plotted in figure (1). After a critical period of time, the lift reaches its quasi-steady value. The aerodynamic functions obtained are in good agreement with the exact Küssner and Sears solutions.

The indicial response of a rectangular section ($\frac{H}{C} = 0.12$), plotted in figure (1), is quite different from the previous indicial response of an airfoil. A strong "overshoot" in lift occurs. An examination of the aerodynamic flowfields, figure (2), indicates that this overshoot is linked to the flow separating from the leading edge (a bluff body, placed in a turbulent flow, distorts the approach flow and a separation from the leading edge develops as time advances). Therefore, for a bluff body, the form of the associated aerodynamic functions is strongly dependent on the creation of separation bubbles. Note that, after a critical period of time, as the lift approaches its quasi-static value, the separated flows from the leading edges merge with the recirculation wake flow region, at the aft end of the cylinder, and a periodic vortex shedding takes place.

The results obtained for the streamlined bridge deck section are different from those obtained for both the airfoil and the rectangular sections. A small overshoot occurs and the value of the admittance can be expected to be close to unity. For bluff sections, with strong overshoot, the use of this value may not be appropriate. In this case, the signature turbulence contributes largely to the buffeting forces and has to be taken into account.

5 Conclusion

In this paper, some examples of CFD calculations have been described. These results provide an alternative to experimental data for deriving aerodynamic functions useful in buffeting analysis. Moreover, they show that the form of the indicial and admittance functions is strongly dependent on the creation of separation bubbles and on the formation of vortices in the wake of the body. Therefore, they provide a simple yet effective way to determine the limitations of the standard quasi-static formulation, commonly used in buffeting analysis, which imply constant unit admittances.

References

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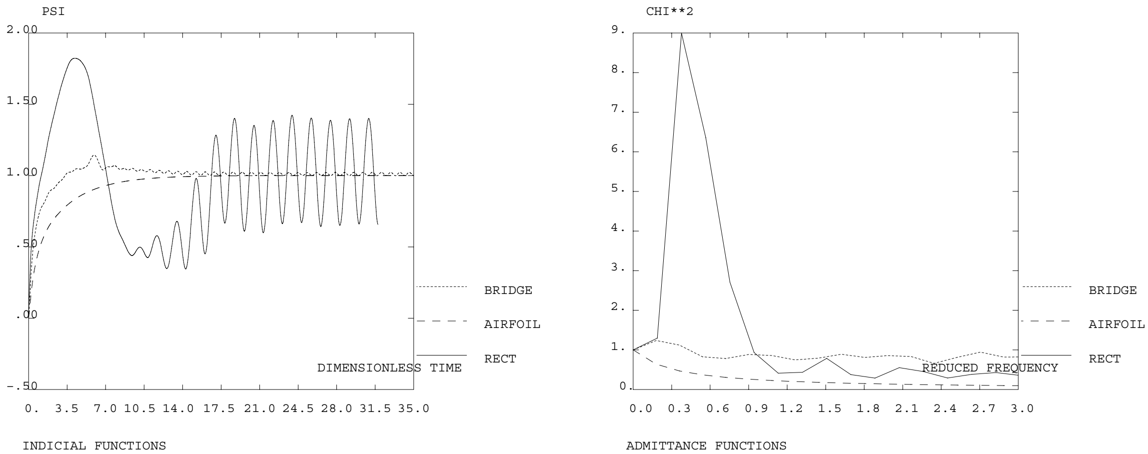

Figure 1: Aerodynamic functions

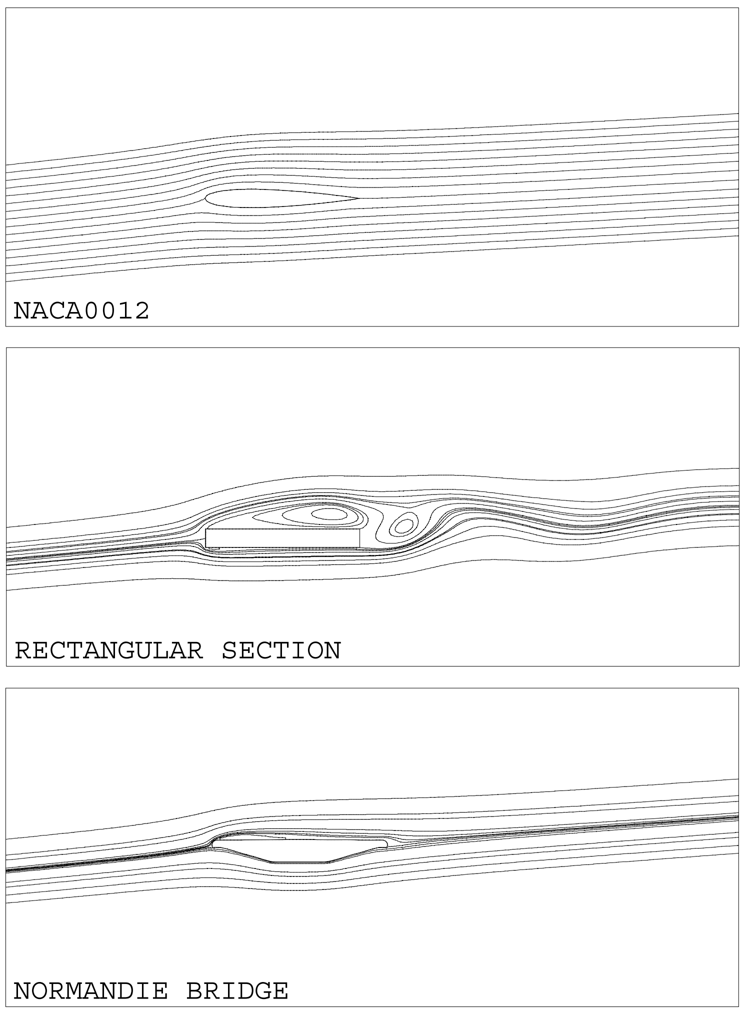

Figure 2: Aerodynamic flowfields